

Chapter 13 Exploring the final frontier

Short investigation 13.2: Circular motion

Aim

To compare uniform circular motion to orbital motion

Materials

Rubber stopper, glass tube, 50 g mass carrier, 50 g slot masses, stopwatch, string, sticky tape, metre rule

Part A: Uniform circular motion

Theory

The expression for the centripetal force in uniform circular motion is $F_c = \frac{mv^2}{r}$. This can be

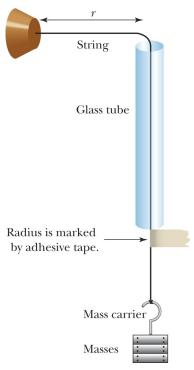
rearranged to show the relationship between v and r: $r = \left(\frac{m}{F}\right)v^2$ or $r \propto v^2$. In words, the

radius of the motion is *directly* proportional to the square of the orbital velocity, as long as the centripetal force is kept constant.

Use this relationship to predict what will happen to the orbital velocity if the radius of the orbit is decreased.

Method

- 1. Record the mass of the rubber stopper being used as a bob.
- 2. Attach the rubber stopper to a length of string approximately 1.5 m long and then thread the loose end of the string through the glass tube.
- 3. Attach the 50 g mass carrier to the loose end of the string as shown in the figure below.



- 4. Place a piece of sticky tape on the string at the point shown in the figure so that the distance *r* is 50 cm.
- 5. Ensure that you have ample room around you, hold the glass tube above your head, and swing the bob around in a horizontal circular motion. The mass carrier will provide the centripetal force to keep the bob moving in its circular path. Adjust your frequency of rotation so that the sticky tape just touches the bottom of the glass tube. This will keep the radius of the bob's orbit steady.
- 6. Record the time for the bob to complete 10 revolutions at a constant speed. Do this three times and then use the average of these as the correct period. Use the radius and period to calculate the orbital velocity of the bob.

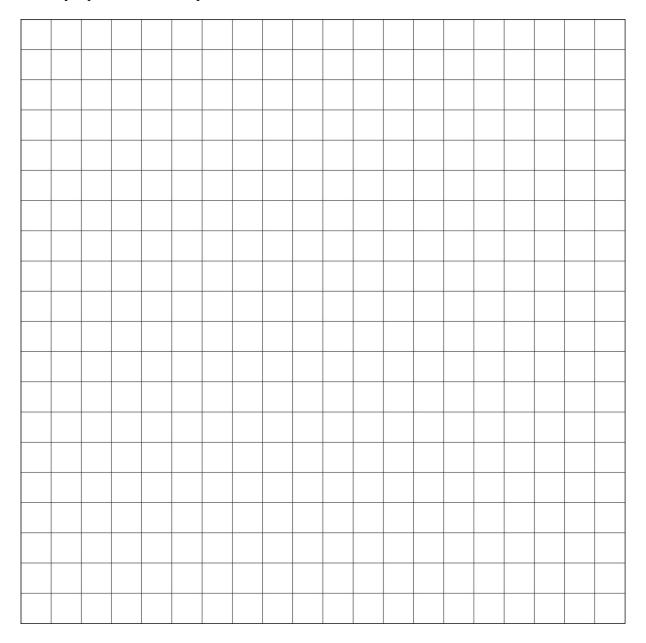
Results

Table 13.2A

Radius, r	Hanging mass (g)	Time for 10 revolutions (s)	Time for 1 revolution (period, T) (s)	Orbital velocity $(= 2\pi r/T)$ (m s ⁻¹)	v² (m s⁻²)
0.5	50				
0.6	50				
0.7	50				
0.8	50				
0.9	50				
1.0	50				

Analysis of results

1. On the grid provided, draw a graph of radius versus orbital velocity squared. The radius was the independent variable so it should be placed on the horizontal axis, and orbital velocity squared should be placed on the vertical axis.



- 2. What happens to the value of the orbital velocity as the radius is increased?
- 3. Describe the relationship shown between radius and orbital velocity for uniform circular motion with a constant centripetal force.

Part B: Orbital motion Theory

In this part we will examine the relationship between the orbital radius and the orbital velocity of a satellite orbiting a planet. Here gravity provides the centripetal force to maintain the circular motion, and this is described by Newton's law of universal gravitation:

$$F_{\rm G} = {\rm G} \, \frac{m_{\rm E} m_{\rm S}}{r^2}.$$

There is a basic difference between this and the standard circular motion seen in part A — the force varies with the radius. As the distance between the Earth and a satellite increases, the gravitational force between them reduces sharply. This is an inverse square relationship, meaning that the force is inversely proportional to the square of the distance between the two objects.

Using Newton's law of universal gravitation it is possible to derive an equation for the orbital velocity of a satellite. This can be used to show the relationship between v and r for a satellite.

$$v = \sqrt{\frac{Gm_E}{r}}$$

$$r = \frac{Gm}{v^2} \text{ or } r \propto \frac{1}{v^2}.$$

Expressed in words, the radius of a satellite's orbit is *inversely* proportional to the square of the orbital velocity, which is quite different to the case in part A above.

Use this new relationship to predict what will happen to the orbital velocity if the radius of the orbit is decreased.

Method

Repeat the procedure used in part A, but this time adjust the hanging masses to the values shown in table 13.2B. (These values have been calculated to simulate the inverse square relationship discussed in the theory above.)

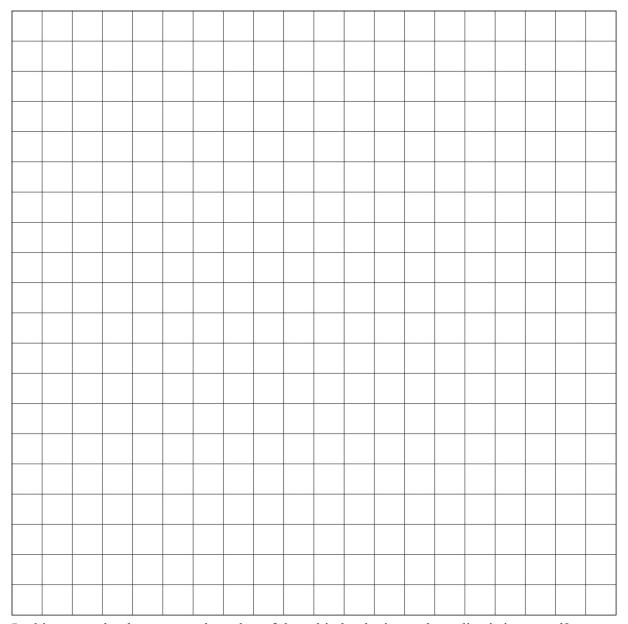
Results

Table 13.2B

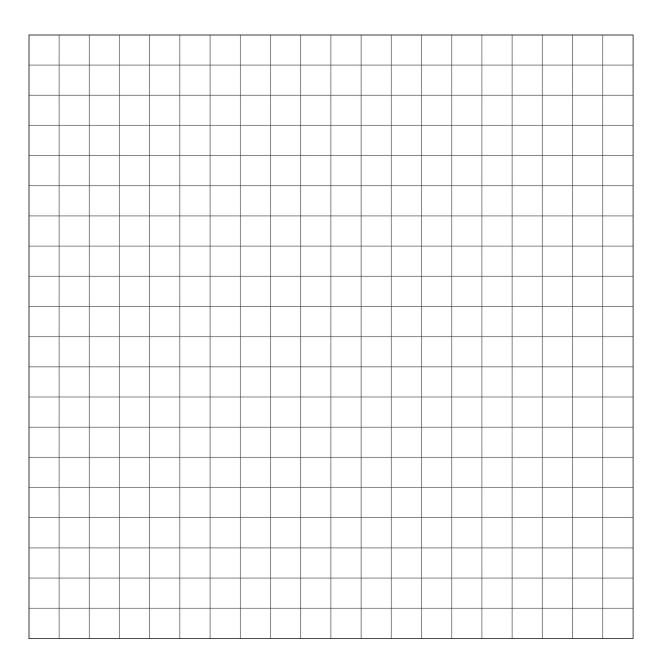
Radius, r	Hanging mass (g)	Time for 10 revolutions (s)	Period, T	Orbital velocity (= 2πr/T) (m s ⁻¹)	v ² (m s ⁻²)	1/v ² (s ² m)
0.5	200					
0.6	140					
0.7	100					
0.8	80					
0.9	60					
1.0	50					

Analysis of results

1. Use the data in table 13.2B to draw a graph of radius versus orbital velocity squared, just as before, on the grid below. This graph should look quite different to the earlier graph.



- 2. In this case, what happens to the value of the orbital velocity as the radius is increased?
- 3. Describe this relationship between radius and orbital velocity for the orbital motion of a satellite.
- 4. In order to examine this relationship further, draw a third graph. This time, use the grid below to draw a graph of radius (still on the horizontal axis) versus $1/v^2$



5. A straight-line graphical relationship is usually seen as confirmation of a relationship. In your judgement, does this last graph confirm that the radius of a satellite's orbit is *inversely* proportional to the square of the orbital velocity?

Conclusion

Write an appropriate conclusion that responds to the stated aim of this investigation.

Notes: